

# **STUDY MATERIAL**

**SUBJECT : MECHANIC OF SOLIDS (MOS)**  
**( MODULE - I & II)**

**SEMESTER : 3<sup>RD</sup>**  
**BRANCH : MECHANICAL ENGINEERING**

## **CONTENTS :**

- OBJECTIVE TYPE QUESTIONS AND ANSWERS
- SHORT TYPE QUESTIONS AND ANSWERS
- LONG TYPE QUESTIONS AND ANSWERS

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**Chapter-3 :Thin Pressure Vessel**

## **BPUT SYLLABUS ( MODULE- I & II )**

### **1. Concept of Stress:**

Load, Stress, Principle of St.Venant, Principle of Superposition, Strain, Hooke's law, Modulus of Elasticity, Stress-Strain Diagrams, Working Stress, Factor of safety, Strain energy in tension and compression, Resilience, Impact loads

**2. Analysis of Axially Loaded Members :** Composite bars in tension and compression ,temperature stresses in composite rods, Concept of Statically indeterminate problems.

Shear stress, Complimentary shear stress, Shear strain, Modulus of rigidity, Poisson's ratio, Bulk Modulus, Relationship between elastic constants.

### **3. Biaxial State of Stress :**

Analysis of Biaxial Stress,Plane stress, Principal plane, Principal stress, Mohr's

### **4. Biaxial State of Strain :**

Two dimensional state of strain, Principal strains, Mohr's circle for strain, Calculation of principal stresses from principal strains, Strain Rosette, Circle for Biaxial Stress.

### **5.Thin Pressure Vessel :**

Stresses in thin cylinders and thin spherical shells under internal pressure, wire winding of thin cylinders.

**DEPARTMENT OF MECHANICAL ENGINEERING**

## **MODULE – I OBJECTIVE TYPE QUESTIONS AND ANSWERS**

1. A material with identical properties in all directions is known as  
(a) homogeneous      (b) isotropic      (c) elastic      (d) none of these
2. The units of stress in the SI system are  
(a) kg/m<sup>2</sup>      (b) N/mm<sup>2</sup>      (c) MPa      (d) any one of these
3. In a lap-riveted joint, the rivets are mainly subjected to \_\_\_\_\_ stress.  
(a) shear      (b) tensile      (c) bending      (d) compressive
4. The resistance to deformation of a body per unit area is known as  
(a) stress      (b) strain  
(c) modulus of elasticity      (d) modulus of rigidity
5. Stress developed due to external force in an elastic material  
(a) depends on elastic constants      (b) does not depend on elastic constants  
(c) depends partially on elastic constants
6. Strain is defined as deformation per unit  
(a) area      (b) length      (c) load      (d) volume
7. Units of strain are  
(a) mm/m      (b) mm/mm      (c) m/mm      (d) no units
8. Hooke's law is valid up to the  
(a) elastic limit      (b) yield point  
(c) limit of proportionality      (d) ultimate point
9. The ratio of linear stress to linear strain is known as  
(a) bulk modulus      (b) modulus of rigidity  
(c) Young's modulus      (d) modulus of elasticity
10. The units of modulus of elasticity are the same as of  
(a) stress      (b) modulus of rigidity      (c) pressure      (d) any one of these
11. The change in length due to a tensile force on body is given by  
(a)  $PL/AE$       (b)  $PLA/E$       (c)  $PLE/A$       (d)  $AE/PL$
12. Approximate Value of Young's modulus for mild steel is  
(a) 100 GPa      (b) 205 MPa      (c) 205 GPa      (d) 100 MPa
13. 1MPa is equal to  
(a) 1N/m<sup>2</sup>      (b) 1N/mm<sup>2</sup>      (c) 1kN/m<sup>2</sup>      (d) 1kN/mm<sup>2</sup>



## *Answers*

- |         |              |              |         |         |         |
|---------|--------------|--------------|---------|---------|---------|
| 1. (b)  | 2. (b and c) | 3. (a)       | 4. (a)  | 5. (b)  | 6. (b)  |
| 7. (d)  | 8. (c)       | 9. (c and d) | 10. (d) | 11. (a) | 12. (c) |
| 13. (b) | 14. (c)      | 15. (b)      | 16. (c) | 17. (a) | 18. (c) |
| 19. (c) | 20. (d)      | 21. (a)      | 22. (b) | 23. (d) | 24. (b) |
| 25. (a) | 26. (b)      | 27. (d)      | 28. (c) | 29. (b) | 30. (a) |
| 31. (b) |              |              |         |         |         |

## SET-1

### Objective Type Questions

1. The strain energy stored in a bar is given by

(a)  $\frac{PL}{AE}$       (b)  $\frac{PL^2}{2AE}$       (c)  $\frac{P^2L}{AE}$       (d)  $\frac{P^2L}{2AE}$

2. Strain energy of a member is given by

(a)  $\frac{\sigma^2}{2E} \times \text{volume}$       (b)  $\frac{P^2L}{2AE}$   
 (c)  $\frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$       (d) all of these

3. Modulus of resilience is

- (a) percentage of elongation of an elastic body  
 (b) strain energy stored in the elastic body  
 (c) strain energy per unit volume of the elastic body

4. Proof resilience is the maximum energy stored at

- (a) plastic limit      (b) limit of proportionality      (c) elastic limit

5. Modulus of toughness is the area of the stress-strain diagram up to

- (a) rupture point      (b) yield point  
 (c) limit of proportionality      (d) none of these

6. Shear strain energy per unit volume is given by

(a)  $\frac{\tau^2}{4G}$       (b)  $\frac{\tau^2}{2G}$       (c)  $\frac{2\tau^2}{3G}$       (d)  $\frac{\tau}{4G}$

7. Strain energy of a bar of conical section is

(a)  $\frac{P^2L}{\pi EdD}$       (b)  $\frac{2PL}{\pi EdD}$       (c)  $\frac{2P^2L}{EdD}$       (d)  $\frac{2P^2L}{\pi EdD}$

8. Strain energy stored in a body due to suddenly applied load compared to when applied slowly is

- (a) twice      (b) four times      (c) eight times      (d) half

Answers:

1. (d)      2. (d)      3. (c)      4. (c)      5. (a)      6. (b)  
 7. (d)      8. (a)

### Objective Type Questions



## *Answers*

1. (a)      2. (d)      3. (c)      4. (b)      5. (c)      6. (b)  
 7. (a)      8. (c)      9. (d)      10. (d)      11. (b)      12. (a)  
 13. (b)      14. (a)      15. (b)      16. (d)

SET - 3

## Objective Type Questions

1. A shell may be termed as *thin* if the ratio of thickness of the wall to the diameter of the shell is less than one to  
 (a) 5 (b) 10 (c) 15 (d) 20

2. In a thin cylinder, the hoop stress is given by  
 (a)  $\frac{pd}{4t}$  (b)  $\frac{pd}{t}$  (c)  $\frac{pd}{2t}$  (d)  $\frac{2pd}{t}$

3. In a thin cylinder, the longitudinal stress is given by  
 (a)  $\frac{2pd}{t}$  (b)  $\frac{pd}{t}$  (c)  $\frac{pd}{2t}$  (d)  $\frac{pd}{4t}$

4. In a thin cylinder, the ratio of hoop stress to longitudinal stress is  
 (a) 1/4 (b) 1/2 (c) 2 (d) 4

5. In a thin spherical shell, the hoop stress is given by  
 (a)  $\frac{pd}{4t}$  (b)  $\frac{pd}{2t}$  (c)  $\frac{pd}{t}$  (d)  $\frac{2pd}{t}$

6. The volumetric strain in a thin spherical shell is  
 (a)  $\frac{3pd}{4tE}(1-v)$  (b)  $\frac{3pd}{4tE}(1-2v)$  (c)  $\frac{pd}{4tE}(1-v)$  (d)  $\frac{pd}{4tE}(1-2v)$

7. The initial hoop stress in a thin cylinder when it is wound with a wire under tension is  
 (a) zero (b) tensile (c) compressive (d) bending

8. In a thick-walled cylinder subjected to internal pressure, maximum hoop stress occurs at  
 (a) outer wall (b) inner wall (c) midpoint of thickness

9. In thick cylindrical pressure vessels, the variation of the hoop stress is  
 (a) parabolic (b) uniform (c) linear (d) cubic

10. In thick cylindrical pressure vessels, the variation of the radial stress is  
 (a) parabolic (b) uniform (c) linear (d) cubic

11. In a thick-walled cylinder subjected to external pressure, the hoop stresses are  
 (a) tensile (b) compressive (c) bending

12. The use of compound tubes subjected to internal pressure are made to  
 (a) even out the stresses (b) increase the thickness  
 (c) increase the diameter of the tube (d) increase the strength

13. In compound tubes, initially, the inside diameter of the outer tube is \_\_\_\_\_ the outside diameter of the inner tube.  
 (a) smaller than (b) larger than (c) equal to (d) 1.2 times

14. The maximum stress in thick cylinders is  
 (a) Radial stress (b) hoop stress (c) longitudinal stress

15. In thick spherical pressure vessels, the variation of the stresses is  
 (a) linear (b) uniform (c) parabolic (d) cubic

Answers

1. (c)      2. (c)      3. (d)      4. (c)      5. ~~(c)~~      6. (a)  
7. (c)      8. (b)      9. (a)      10. (a)      11. (b)      12. (a)  
13. (a)      14. (b)      15. (d)

# Strength Of Materials

## MODULE-1 & 2

### SHORT TYPE QUESTIONS AND ANSWERS

Q.1. Define 'stress'. How it is expressed?

Ans: When a material is subjected to an external force, a resisting force is set up within the component. The internal resisting force per unit area of acting on a material or intensity of force distributed over a given section is called the 'stress' at a point.

- Stress is expressed as, stress,  $\sigma = \frac{\text{Resisting Force}}{\text{Area}} = \frac{P}{A}$

So, the unit through which stress is expressed are  $\text{MN/m}^2$  (or MPa),  $\text{N/m}^2$  (Pa),  $\text{KN/m}^2$  (or KPa) or  $\text{N/mm}^2$

Q.2. Define 'true stress' and 'true strain'.

Ans: True stress :- True stress is the load at any elongation divided by the cross-sectional area at that elongation.

$$\text{True stress} = \frac{\text{Force or load}}{\text{Cross-section area existing at the instant being considered (or, instantaneous area)}} \\ = \sigma (1 + \epsilon)$$

True strain : True strain is the change in length with reference to the <sup>instantaneous</sup> gauge length rather than the original length.

$$\text{True strain} = \int_{L_0}^L \frac{dL}{L} = \ln\left(\frac{L}{L_0}\right) = \ln(1 + \epsilon)$$

Q.3. Define proof stress.

Ans: Proof stress is the stress at which the stress-strain curve departs from a straight line by not more than 0.1% of length of the test piece. The material is said to have passed the proof stress if application of certain load for 15 seconds does not produce more than 0.1% elongation.

Q.4: What is the function of 'extensometer' in stress-strain measurement?

Ans:- An extensometer is an instrument or device by which the changes in length of specimen under test can be precisely measured.

Q.5: Define Hooke's Law. State its validation for a material

Ans: Hooke's law states that within elastic limits, the stress ( $\sigma$ ) is directly proportional to the strain ( $\epsilon$ ).

Mathematically,  $\sigma \propto \epsilon$

$$\Rightarrow \boxed{\sigma = E\epsilon}$$

where 'E' is the constant of proportionality and is called as 'Young's modulus' or 'modulus of elasticity'.

- Hooke's law is valid upto elastic limit (or proportional limit) of material of a specimen.

Q.6 Define 'modulus of elasticity'?

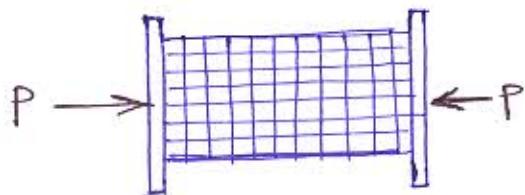
Ans: Modulus of elasticity (E) :- Below proportional limit (or elastic limit in some other books), stress and strain are related to one another by a constant of proportionality known as 'modulus of elasticity'.

Thus, Modulus of elasticity or Young's modulus,  $E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon}$

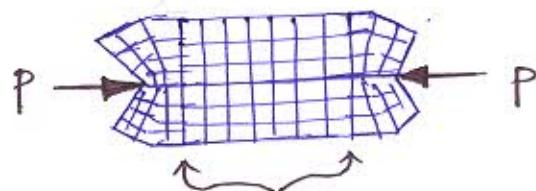
- 'E' represents the slope of the stress-strain curve (only the slope of the initial straight line portion of the plot)
- 'E' represents 'resistance to elastic deformation'. Resistance to elastic deformation is more commonly called 'stiffness'.

Q.7. State Saint-Venant's principle.

Ans: According to this principle, the distribution of internal stresses or strains on a section of a body, which are at sufficient distance from the surface of the load application, is not affected by the nature of actual application of load over the surface.



(Fig-a)



(fig-b) Remain unaffected by the application of external load (P)

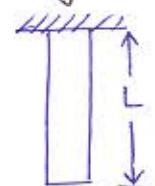
Q.8. State principle of superposition.

Ans: The principle of superposition states that if a body is acted upon by a number of loads on various segments of a body, then the net effect on the body is the sum of the effect caused by each of the loads acting independently on the respective segment of the body. Thus, each segment can be considered for its equilibrium. This is done by drawing 'free body diagrams' of each segments.

Q.9. Write the expression for elongation of a bar having (i) rectangular section, (ii) conical section, due to its self-weight.

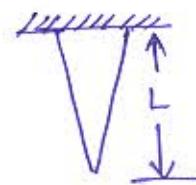
Ans : (i) Rectangular Section

$$\text{Elongation of the bar, } \Delta L = \frac{WL}{2AE}$$



(ii) Conical section

$$\text{Elongation of the bar, } \Delta L = \frac{WL}{6AE}$$



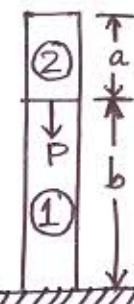
where,

'W' = weight of the whole bar

Q.10. Determine the displacement of the free end of the bar by taking into account the weight of the bar, the specific weight of material  $\gamma$ .

Ans: Compression of part-1 due to load 'P',  $(\Delta L)_1 = \frac{Pb}{AE}$

Compression of bar due to selfweight,  $(\Delta L)_2 = \frac{\gamma(a+b)^2}{2E}$



$\therefore$  Displacement of free end,  $\Delta L = (\Delta L)_1 + (\Delta L)_2$

$$\Rightarrow \boxed{\Delta L = \frac{Pb}{AE} + \frac{\gamma(a+b)^2}{2E}}$$

Q.11. Calculate the end reactions at the supports.

Solution: As the total elongation is to be ZERO, since both ends are fixed;

i.e.,  $\Delta L = 0$

$$\Rightarrow \frac{RL_1}{AE} + \frac{(R-10)L_2}{AE} = 0$$

$$\Rightarrow (R \times 1) + (R-10) \times 2 = 0$$

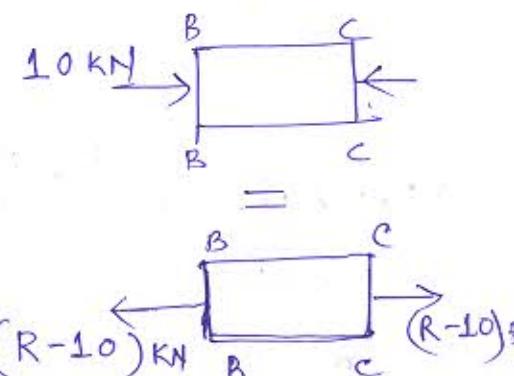
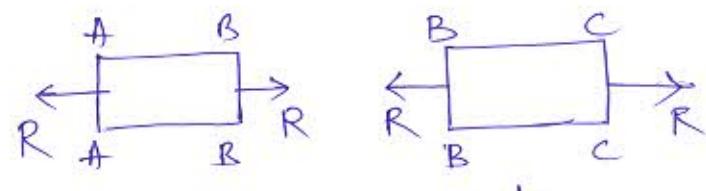
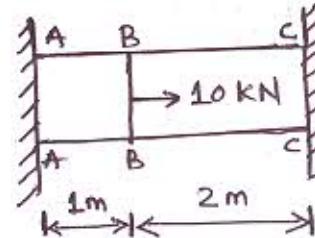
$$\Rightarrow R + 2R - 20 = 0$$

$$\Rightarrow \boxed{R = \frac{20}{3} \text{ kN}} \text{ and}$$

$$R-10 = \frac{20}{3} - 10 = -\frac{10}{3} \text{ kN}$$

$$\therefore R_A = R = \frac{20}{3} \text{ kN } (\text{Tensile})$$

$$\text{and } R_C = R-10 = -\frac{10}{3} \text{ kN } (\text{compressive})$$



Q.12. Define Poisson ratio.

Ans: Poisson ratio ( $\mu$ ) : It is the ratio of lateral strain to the longitudinal strain.

$$\text{Poisson's ratio, } \mu = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

- The value of ' $\mu$ ' ranges from 0.25 to 0.33.
- For most of the engineering materials,  $0 \leq \mu \leq 0.5$

N.B.

Lateral and Longitudinal strain:- The strain produced in the direction of load is known as Longitudinal strain and the strain produced perpendicular to load application is known as Lateral strain.

Q.13. For a cube made of cork, the ratio of Young's modulus and modulus of rigidity is \_\_\_\_\_.

Ans: For cork,  $\mu = 0$

$$E = 2G(1 + \mu)$$

$$\Rightarrow \frac{E}{G} = 2(1 + \mu) = 2(1 + 0) = 2$$

Ans

Q.14. A rod tapers uniformly from 50 mm to 20 mm diameter in a length of 500 mm. If the rod is subjected to an axial load of 10 kN, find the extension in the rod.

Assume  $E = 2 \times 10^5 \text{ MPa}$ .

Solution:- Extension in varying cross-section or taper rod is

$$\text{given by, } \Delta L = \frac{4PL}{\pi D d E} = \frac{4 \times 10 \times 10^3 \times 0.5}{\pi \times 0.05 \times 0.02 \times 2 \times 10^5}$$

$$\Rightarrow \Delta L = 3.1830 \times 10^{-5} \text{ m} = 0.0318 \text{ mm}$$

Ans

Q.15. What do you mean by 'compound bar'?

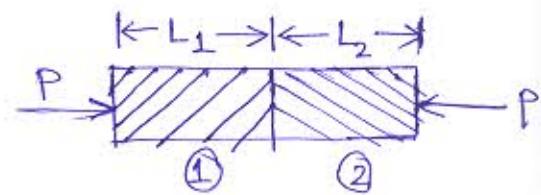
Ans:- Compound bar :- Any tensile or compressive member which consists of two or more bars/tubes/members in parallel, usually of different material, is called a 'compound bar'.

Q.1

Q.16. Define composite bar.

Ans : Any tensile or compressive member which consists of two or more bars in series, usually of different materials, is called composite bars.

- In this case, load ( $P$ ) on both the rods will be same but strain produced will be different.



$$P = A_1 E_1 \epsilon_1 = A_2 E_2 \epsilon_2$$

$$\text{Total strain, } \epsilon = \frac{P}{A_1 E_1} + \frac{P}{A_2 E_2}$$

Q.17. Define temperature (or thermal) stress.

Ans :- When the temperature of a material is changed, its dimensions change. If this change in dimensions is prevented, then a stress is set up in the material which is called 'temperature stress' or 'thermal stress'.

Q.18. Define volumetric strain. Write down the expression of volumetric strain for a cube subjected to three dimension hydrostatic pressure.

Ans : Volumetric strain,  $\epsilon_{\text{vol.}} = \frac{\text{Change in volume}}{\text{Original volume of an object}} = \frac{\Delta V}{V}$

$$\epsilon_{\text{vol.}} = \epsilon_1 + \epsilon_2 + \epsilon_3 = \frac{1}{E} [\sigma_1 + \sigma_2 + \sigma_3 - 2\mu(\sigma_1 + \sigma_2 + \sigma_3)]$$

$$\Rightarrow \boxed{\epsilon_{\text{vol.}} = \frac{1-2\mu}{E} (\sigma_1 + \sigma_2 + \sigma_3)} \quad (\text{for cube})$$

Q.19. Explain Working stress, Allowable stress and factor of safety

Ans: Working stress: Working stress is defined as the actual stress of a material under a given loading.

Allowable stress:- The maximum safe stress that a material can carry is termed as the allowable stress. The allowable stress should be limited to values not exceeding the proportional limit.

Factor of safety:- Since the proportional limit is difficult to determine accurately, the allowable stress is taken as either the yield point (for ductile materials) or ultimate strength (for brittle materials) divided by a factor of safety.

So, the ratio of ultimate (or yield strength) to allowable strength is called the factor of safety.

Q.20. Write down the relationship between E, G and K.

Ans:

$$E = \frac{9KG}{3K + G}$$

where E = Young's modulus  
G = Modulus of rigidity.  
K = Bulk modulus.

Q21. For a given material, Young's modulus is  $110 \text{ GN/m}^2$  and shear modulus is  $42 \text{ GN/m}^2$ . Find the bulk modulus and lateral contraction of a round bar of 37.5 mm diameter and 2.4 m long when stretched by 2.5 mm.

Solution:- Given data:  $E = 110 \text{ GN/m}^2$ ,  $G = 42 \text{ GN/m}^2$

$$\text{We know, } E = 2G(1+\mu) \Rightarrow \mu = \frac{E}{2G} - 1$$

$$= \frac{110}{2 \times 42} - 1 = 0.3$$

$$\text{Also, } E = 3K(1-2\mu) \Rightarrow K = \frac{E}{3(1-2\mu)} = \frac{110}{3(1-2 \times 0.3)} \\ = 91.66 \text{ GPa} \quad \underline{\text{Ans}}$$

We know,  $\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(\frac{\delta D}{D})}{(\frac{\delta L}{L})} = \frac{\delta D \times L}{\delta L \times D}$

$$\Rightarrow \delta D = \frac{\mu \times \delta L \times D}{L} = \frac{0.3 \times 2.5 \times 37.5}{2.4 \times 10^3} = 0.0117 \text{ mm} \quad \underline{\text{Ans}}$$

Q.22. Define resilience and proof resilience.

Ans: Resilience :- It is the total strain energy stored in a body. It can be defined as the ability of a material to regain its original shape on the removal of the applied load.

Proof resilience :- The maximum strain energy which can be stored in a body, is known as proof resilience.

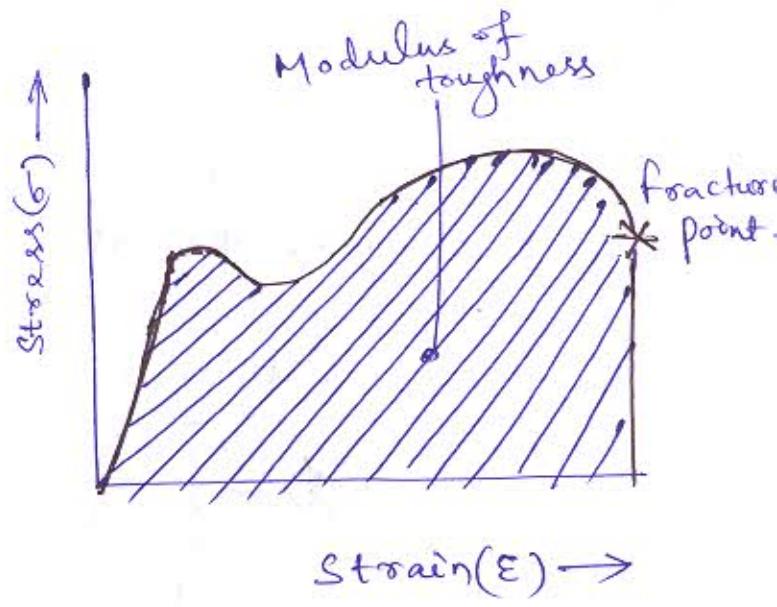
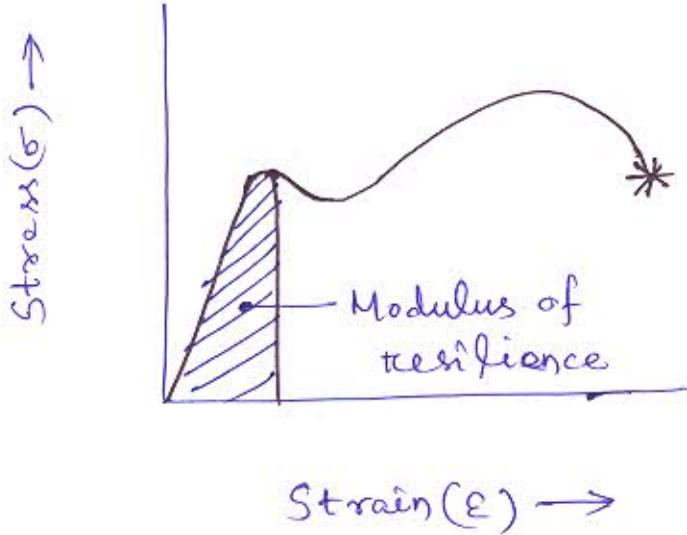
Q.23. Define modulus of resilience and modulus of toughness.

Ans: Modulus of resilience :- The proof resilience per unit volume of a material is known as "modulus of resilience". This happens when the body is stressed upto the elastic limit.

- Modulus of resilience is the area under the stress-strain diagram upto the elastic limit.

Modulus of toughness :- It is the strain energy per unit volume required to cause the material to rupture. Thus, it is a measure of the ~~the~~ ability of a material to absorb energy before fracture.

- It is the area under the stress-strain diagram upto the rupture point.



Q.24. Write down the expressions of strain energy under axial loading, shear loading, torsion and bending.

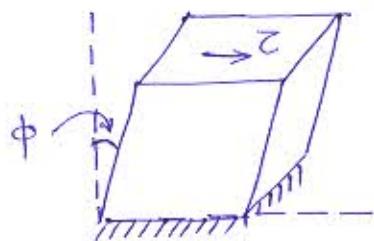
Ans:-

$$\text{Strain energy under axial loading, } U = \frac{P^2 L}{2AE}$$

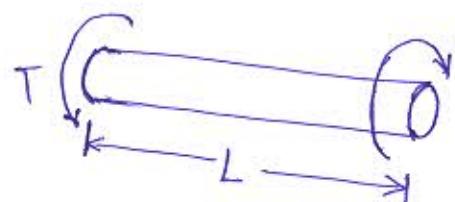
$$\text{Strain energy under shear loading, } U = \frac{\tau^2}{2G} \times \text{volume}$$

$$\text{Strain energy in torsion, } U = \frac{\tau^2}{4G} \times \text{volume or } \frac{T^2 L}{2GJ}$$

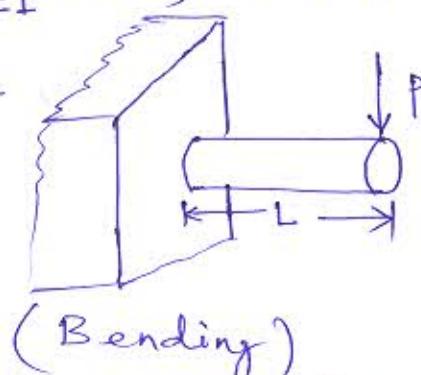
$$\text{Strain energy in bending, } U = \int \frac{M^2}{2EI} dx, \quad M = P \times L$$



(Shear Loading)



(Torsion)



(Bending)

Q.25. An axial pull of 20 KN is applied on a steel rod of 2.5 m long and  $1000 \text{ mm}^2$  in cross-section. Calculate the strain energy stored in the rod. Take,  $E = 200 \text{ GPa}$ .

Solution:- Volume,  $V = A \cdot L = 1000 \times 2.5 \times 10^3 = 2.5 \times 10^6 \text{ mm}^3$

$$\text{Stress in the bar, } \sigma = \frac{P}{A} = \frac{20}{1000} = 0.02 \text{ KN/mm}^2$$

$\therefore$  Strain energy that can be stored in the rod,

$$U = \frac{\sigma^2}{2E} \times V = \frac{(0.02 \times 10^3)^2}{2 \times 200 \times 10^9} \times 2.5 \times 10^6$$

Q26. Strain energy in the rod,  $U = \frac{P^2 L}{2AE} = ?$

Q.26. Determine the strain energy of a prismatic cantilever beam AB.

Solution:- The B.M. at a distance 'x' from end 'A' is  $M_x = -P \cdot x$ .

$$\therefore \text{Strain energy, } U = \int_0^L \frac{M_x^2}{2EI} dx = \int_0^L \frac{(-P \cdot x)^2}{2EI} dx \\ = \left[ \frac{P^2 x^3}{6EI} \right]_0^L$$



$$\Rightarrow \boxed{U = \frac{P^2 L^3}{6EI}}$$

Q.27. A flat ribbon of spring steel 3.2 MM wide and 0.5 MM thick is wound round a cylinder 50 cm in diameter. Find the maximum stress in the steel ribbon and the energy in bending stored ( $E = 220 \text{ GPa}$ ) per metre length of the ribbon.

Solution:- We know, bending equation  $\frac{\sigma}{y} = \frac{E}{R}$

$$\Rightarrow \sigma = \frac{E}{R} \times y = \frac{220 \times 10^9 \times 0.025}{25} \\ = 220 \text{ MPa.}$$

$\therefore$  Strain energy stored per metre length ( $L=1\text{m}$ ) of ribbon

is given by,  $U = \frac{\sigma^2}{6E} \times \text{Volume}$

$$= \frac{(220 \times 10^6)^2}{6 \times 220 \times 10^9} \times 3.2 \times 0.5 \times 1 \times 10^{-6} \\ = 58.67 \times 10^{-3} \text{ N-m}$$

or 58.67 joule

Ane

**Q.28.** What do you mean by 'principal planes' and 'principal stresses'?

Ans: In general, a body may be acted upon by direct stresses and shear stresses. However, it will be seen that even in such complex systems of loading, there exist three mutually perpendicular planes, on each of which the resultant stress is wholly normal (i.e., perpendicular). These planes are known as 'principal planes' and the normal stress across these principal planes, is known as 'principal stresses'.

**Q.29.** What is Mohr's stress circle? How is it useful in the solution of stress-analysis problems?

Ans: Mohr's stress circle is a graphical plot between direct stress (taken along abscissa) and shear stress (taken along ordinate),

- It is used to determine the stress components on any given inclined plane under axial loading (uniaxial, bi-axial or tri-axial loading)

**Q.30.** What do you mean by 'strain rosette'?

Ans:- Strain in any direction can be measured by using an instrument known as 'strain gauge'. When a set of strain gauges (usually 3 strain gauge) is used to determine the strain in different directions, it is known as 'strain rosette'.

## Strength Of Materials

### MODULE-1 & 2

#### LONG TYPE QUESTIONS AND ANSWERS

Problem-1: A round bar as shown below, is subjected to an axial tensile load of 100 kN. What must be the diameter 'd' if the stress there is to be 100 MN/m<sup>2</sup>? Also, find the total elongation, take E = 200 GPa.

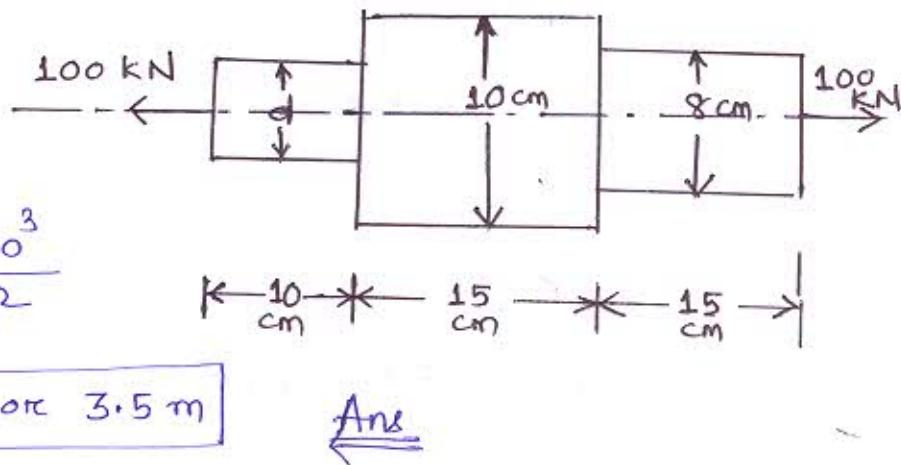
Solution:  $d = ?$

$$\text{Stress, } \sigma = \frac{P}{A}$$

$$= \frac{P}{\left(\frac{\pi d^2}{4}\right)}$$

$$\Rightarrow 100 \times 10^6 = \frac{100 \times 10^3}{\frac{\pi \times d^2}{4}}$$

$$\Rightarrow \boxed{d = 0.035 \text{ m or } 3.5 \text{ m}}$$



Ans

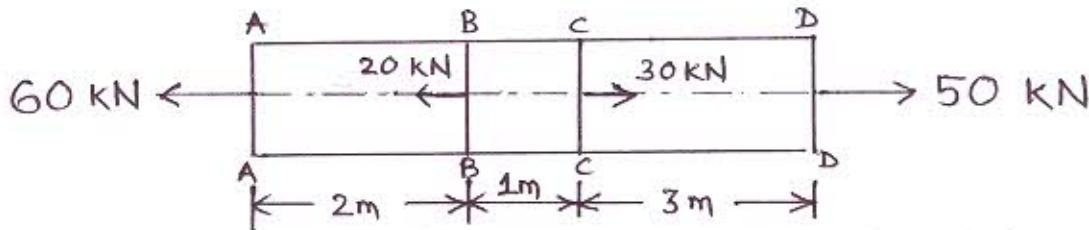
Total Elongation,  $\Delta L$ :

$$\Delta L = P \left( \frac{L_1}{A_1 E} + \frac{L_2}{A_2 E} + \frac{L_3}{A_3 E} \right)$$

$$= \frac{100 \times 10^3}{200 \times 10^9} \left[ \frac{0.10}{\frac{\pi}{4} (0.035)^2} + \frac{0.15}{\frac{\pi}{4} (0.10)^2} + \frac{0.15}{\frac{\pi}{4} (0.08)^2} \right]$$

$$\Rightarrow \boxed{\Delta L = 0.0745 \text{ mm}} \quad \text{Ans}$$

Problem-2: A steel bar of 25 mm diameter is acted upon by forces as shown in figure below. What is the total elongation of the bar? Take  $E = 190 \text{ GPa}$ .

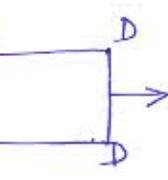
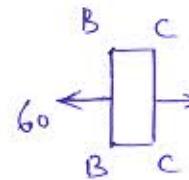
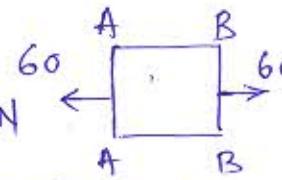


Solution: Calculation of forces in various segments

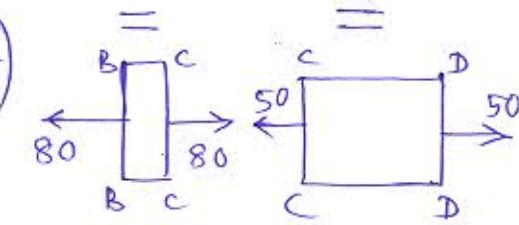
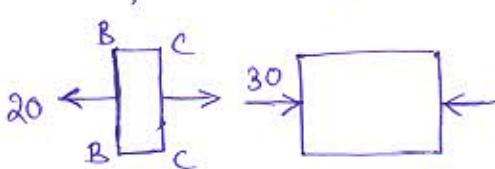
Forces in various segments are considered by taking Free body diagrams of each segment as follows:

Segment AB:

$$\begin{aligned} \text{Force at section A-A} &= 60 \text{ kN} \\ &= \text{Force at section B-B} \end{aligned}$$



+



Segment BC:

$$\text{Force at section B-B} = \text{force at sec B-B}$$

$$\begin{aligned} &+ \\ &20 \text{ kN (applied tensile)} \\ &= 80 \text{ kN (tensile)} \\ &= \text{force at section C-C} \end{aligned}$$

(Fig - Free Body Diagrams)

Segment CD:

$$\text{Force at section C-C} = 80 \text{ kN tensile (as above)}$$

$$- 30 \text{ kN (applied compressive force at C-C)}$$

$$= 50 \text{ kN (tensile)}$$

$$= \text{force at section D-D}$$

Determination of Total Elongation ( $\Delta L$ ):

$$\Delta L = \sum_{i=1}^3 \frac{P_i L_i}{AE} = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE}$$

$$= \frac{1}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^9} \left[ (60 \times 10^3 \times 2 \times 10^3) + (80 \times 10^3 \times 1 \times 10^3) + (50 \times 10^3 \times 3 \times 10^3) \right]$$

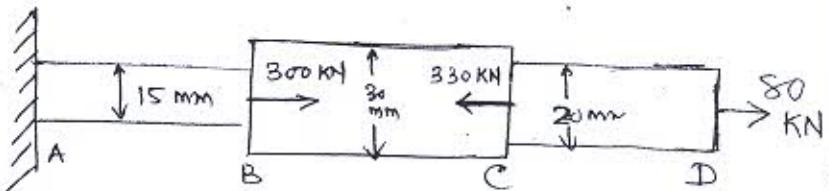
$$= 3.75 \text{ mm}$$

Ans

Problem-3: A steel circular bar has three segments. Determine  
 (i) the total elongation of the bar  
 (ii) the length of the middle segment to have zero elongation of the bar.  
 (iii) the diameter of the last segment to have zero elongation of the bar.  
 Take  $E = 205 \text{ GPa}$ .

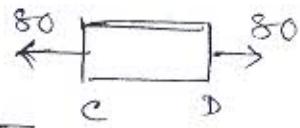
Solution:

Calculation of Forces in Various Segments



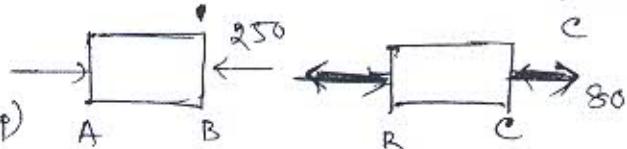
(\* Consider FBD of segment from RHS)  
 $\hookrightarrow$  LHS is fixed.

Segment CD: Force at section D-D = 80 KN  
 = Force at section C-C.



Segment BC:

$$\begin{aligned} \text{Force at section C-C} &= 80 - 330 \\ &= -250 \text{ KN (Comp)} \end{aligned}$$



Segment AB

= Force at sec-B-B

+

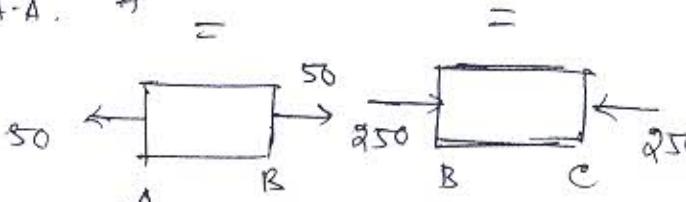
+

$$\begin{aligned} \text{Force at section B-B} &= -250 + 300 \\ &= 50 \text{ KN (tensile)} \end{aligned}$$

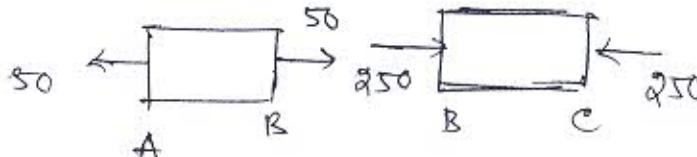
= Force at sec-A-A.

=

=



① Total Elongation ( $\Delta L$ )



$$\Delta L = \frac{1}{\frac{\pi}{4} \times 205 \times 10^3} \left[ \frac{80 \times 10^3 \times 250}{(20)^2} + \frac{-250 \times 10^3 \times 20}{(30)^2} + \frac{50 \times 10^3 \times 150}{(15)^2} \right]$$

$$= 0.173 \text{ mm. Ans}$$

② Length of middle segment to have zero elongation ( $L$ )

$$\Delta L = \frac{1}{\frac{\pi}{4} (205 \times 10^3)} \left[ \frac{80 \times 10^3 \times 250}{(20)^2} + \frac{-250 \times 10^3 \times L}{(30)^2} + \frac{50 \times 10^3 \times 150}{(15)^2} \right]$$

$$\Rightarrow L = 200 \text{ mm}$$

$$= 0$$

Total elongation of the bar = 1 mm  $(1.25 \text{ m} - 1 \text{ mm})$

$$\Rightarrow \frac{(100-P) \times 10^3 \times 1.25}{4 \times 10^{-4} \times 200 \times 10^9} + \frac{-P \times 10^3 \times 1.199}{5 \times 10^{-4} \times 200 \times 10^9} = 1 \text{ mm}$$

$$\Rightarrow (15.625 - 0.28125P) \times 10^{-4} \text{ m} = 1 \times 10^{-3} \text{ m}$$

$$\Rightarrow P = 20 \text{ kN}$$

Hence,  $F_{AB} = 100 - P = 100 - 20 = 80 \text{ kN}$  (tensile)

$$F_{BC} = P = 20 \text{ kN}$$
 (compressive)

and

$$\sigma_{AB} = \frac{80 \times 10^3}{4 \times 10^{-4}} = 200 \text{ MPa}$$
 (tensile)

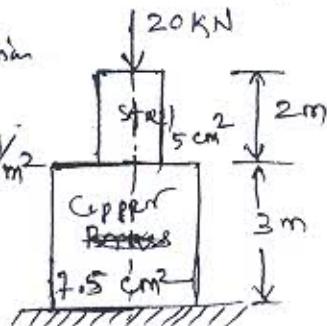
$$\sigma_{BC} = \frac{20 \times 10^3}{5 \times 10^{-4}} = 40 \text{ MPa}$$
 (compressive)

Problem-7: Determine the total compression of the bar shown in figure.

$$E_{\text{Steel}} = 210 \text{ GN/m}^2, E_{\text{Copper}} = 84 \text{ GN/m}^2$$

Sol)

$$\begin{aligned} \Delta L &= (\Delta L)_{\text{Steel}} + (\Delta L)_{\text{Copper}} \\ &= \frac{20 \times 10^3 \times 2}{5 \times 10^{-4} \times 210 \times 10^9} + \frac{20 \times 10^3 \times 3}{2.5 \times 10^{-4} \times 84 \times 10^9} \\ &= 1.333 \text{ mm} \end{aligned}$$



Ans

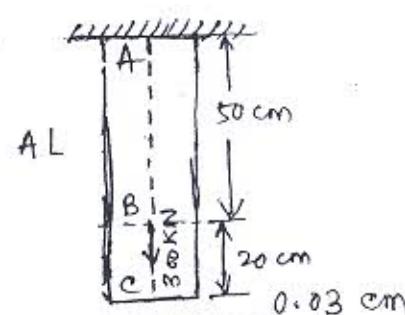
Problem-8:  $A_{AI} = 12.5 \text{ cm}^2, E_{AI} = 70 \text{ GN/m}^2$

$$A_S = 25 \text{ cm}^2, E_S = 210 \text{ GN/m}^2$$

Solution: Elongation of AB under 300 kN load

$$= \frac{300 \times 10^3 \times 50 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9}$$

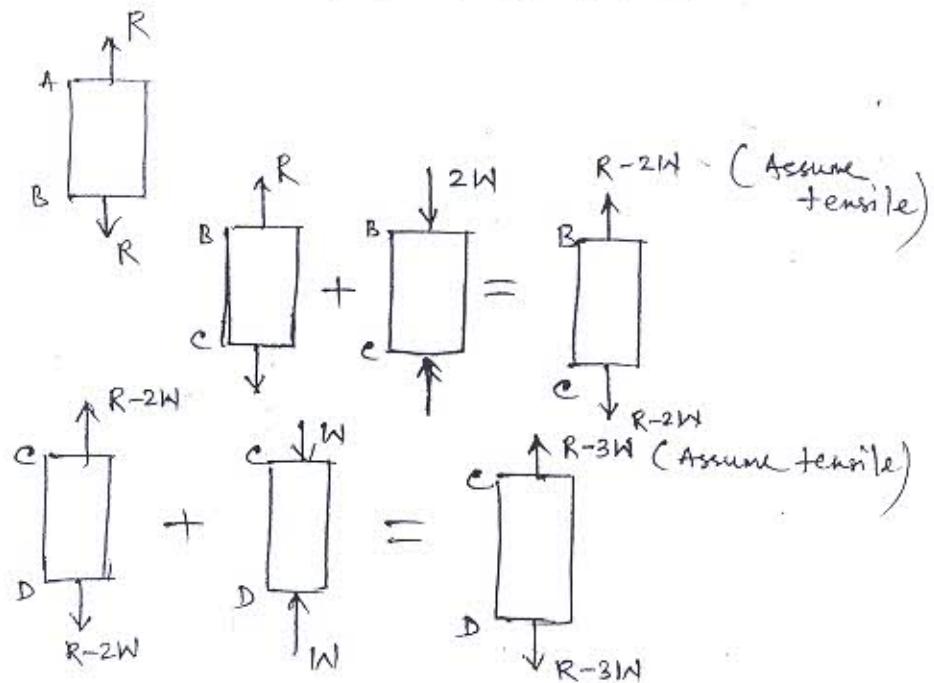
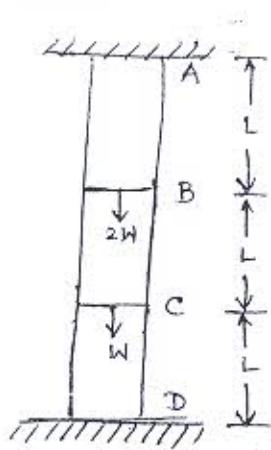
$$= 0.171 \text{ cm} > 0.03 \text{ cm}$$



Hence bars DE and BC will be subjected to compression whereas AB will remain in tension.

Let 'P' be the compressive force in BC and DE, then the

Problem-4: Determine the end reactions if  $W = 1.5 \text{ KN}$



End Reactions: As the total elongation/extension is to be zero since both ends are fixed, i.e.  $\Delta L = 0$

$$\Rightarrow \frac{RL}{AE} + \frac{(R-2W)L}{AE} + \frac{(R-3W)L}{AE} = 0$$

$$\Rightarrow R + R - 2W + R - 3W = 0$$

$$\Rightarrow 3R = 5W$$

$$\Rightarrow R = \frac{5W}{3} = \frac{5 \times 1.5}{3} = 2.5 \text{ KN (upward)} \quad (\text{tensile})$$

and  $R - 3W = 2.5 - (3 \times 1.5) = -2 \text{ KN (upward)}$

Problem-5: Calculate the end reactions at the supports.

Solution: As the total elongation is to be zero, since both ends are fixed, i.e.  $\Delta L = 0$

$$\Rightarrow \frac{RL_1}{AE} + \frac{(R-10)L_2}{AE} = 0$$

$$\Rightarrow R \times 1 + (R-10) \times 2 = 0$$

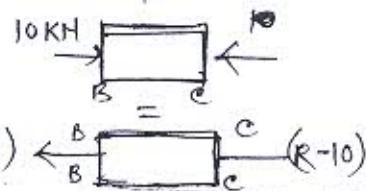
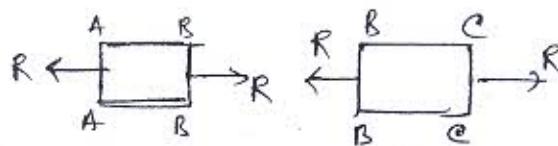
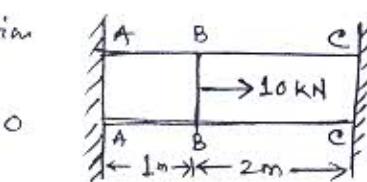
$$\Rightarrow R + 2R - 20 = 0$$

$$\Rightarrow 3R = 20$$

$$\Rightarrow R = \frac{20}{3} \text{ KN}$$

$$R_A = R = \frac{20}{3} \text{ KN}$$

$$R_C = R - 10 = \frac{20}{3} - 10$$

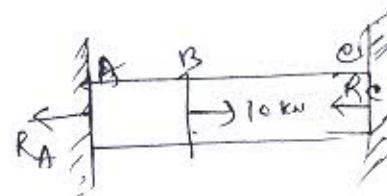


\* Alternate Approach:- Since the supports are rigid, therefore  
 Elongation of segment AB = Compression of segment BC

$$\Rightarrow \frac{R_A L_1}{A \cdot E} = \frac{R_C L_2}{A \cdot E}$$

$$\Rightarrow \frac{R_A \times 1}{A \cdot E} = \frac{R_C \times 2}{A \cdot E}$$

$$\Rightarrow R_A = 2 R_C$$



$$R_A + R_C = 10 \text{ kN}$$

Again  $R_A + R_C = 10 \text{ kN}$

$$\Rightarrow 2R_C + R_C = 10 \text{ kN}$$

$$\Rightarrow R_C = \frac{10}{3} \text{ kN}$$

(Compressive)

$$\text{and } R_A = 2 \times \frac{10}{3} = \frac{20}{3} \text{ kN}$$

(Tensile)

Ans

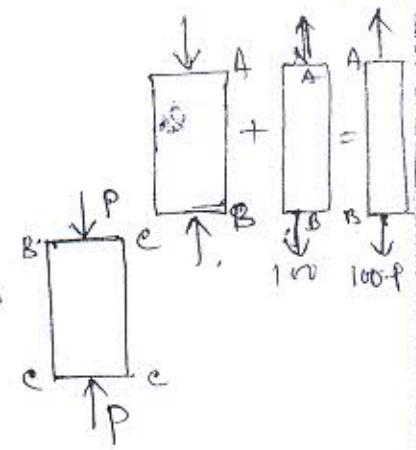
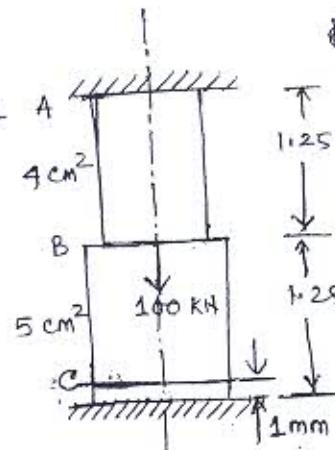
Ques

Problem-6 : Determine ~~the~~

- ① Reaction produced by the support
- ② Force in each segment
- ③ Stress in each segment.

Solution:

In the absence of horizontal rigid support, the portion AB of the bar would elongate by an amount,  $(\Delta L)_{AB} = \frac{PL}{AE} = \frac{100 \times 10^3 \times 1.25 \times 10^3}{4 \times 10^9 \times 200 \times 10^{-9}} = 1.5625 \text{ mm}$



Whereas the lower portion BC of the bar would have remained unaltered. Due to the presence of the horizontal rigid support, the bar AC can move downward by 1 mm only. Since the extension of the bar is ~~more than~~ more than 1 mm, therefore, the bar is subjected to an upward reaction (compressive).

Let the upward reaction be 'P' kN. (Compressive)

Displacement of section  $x-x$  due to self-weight of bar  
of length 'a',  $(\Delta L)_2 = \frac{\gamma \cdot a^2}{2E}$

Displacement of section  $x-x$  due to self-weight of bar  
of length 'b',  $(\Delta L)_3 = \frac{(\gamma \cdot A b) \times a}{A E}$

$$= \frac{PL}{AE}$$

$$\begin{aligned} \text{Total Displacement, } \Delta L &= (\Delta L)_1 + (\Delta L)_2 + (\Delta L)_3 \\ &= \frac{P \cdot a}{AE} + \frac{\gamma \cdot a^2}{2E} + \frac{(\gamma \cdot A b) \cdot a}{AE} \end{aligned}$$

$$\Rightarrow \boxed{\Delta L = \frac{\gamma \cdot a^2}{2E} + \frac{(P + \gamma \cdot A b) \cdot a}{AE}}$$

Problem-7: The stresses on two perpendicular plane through a point in a body are 160 MPa and 100 MPa, both compressive along with a shear stress of 80 MPa. Determine

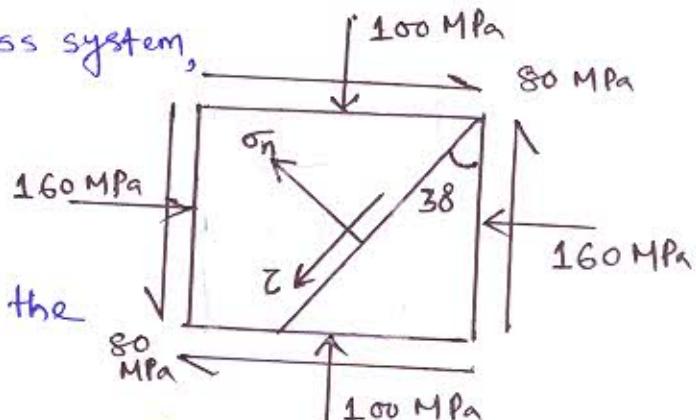
- The normal and the shear stresses on a plane inclined at  $30^\circ$  to the plane of 160 MPa stress. Find also the resultant stress and its direction.
- principal stresses and maximum shear stress.

Solution: A bi-axial and shear stress system,

$$\sigma_x = -160 \text{ MPa}$$

$$\sigma_y = -100 \text{ MPa}$$

$$\tau_{xy} = 80 \text{ MPa}$$



- On a plane inclined at  $30^\circ$  to the plane of 160 MPa stress,

$$\begin{aligned} \text{Normal stress, } \sigma_n &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-160 - 100}{2} + \frac{-160 + 100}{2} \cos 60^\circ + 80 \sin 60^\circ \\ &= -75.7 \text{ MPa} \end{aligned}$$

Ans

$$\begin{aligned} \text{Shear stress, } \tau &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{-160 + 100}{2} \sin 60^\circ + 80 \cos 60^\circ \\ &= 66 \text{ MPa (clockwise)} \end{aligned}$$

Ans

$$\begin{aligned} \text{Resultant stress, } \sigma_R &= \sqrt{\sigma_n^2 + \tau^2} \\ &= \sqrt{(-75.7)^2 + (66)^2} \\ &= 100.4 \text{ MPa} \end{aligned}$$

Ans

$$\begin{aligned} \text{Direction of resultant : } \tan \phi &= \frac{\tau_{30}}{\sigma_{30}} = \frac{66}{75.7} = 0.872 \\ \Rightarrow \phi &= 41.1^\circ \end{aligned}$$

Ans

(P.T.O)

$$\begin{aligned}
 \text{(ii) Principal stresses, } \sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
 &= \frac{-160 - 100}{2} \pm \sqrt{\left(-\frac{160 + 100}{2}\right)^2 + (80)^2} \\
 &= -130 \pm 85.44
 \end{aligned}$$

∴ Major principal stress,  $\sigma_1 = -130 + 85.44 = -44.56 \text{ MPa}$

Minor principal stress,  $\sigma_2 = -130 - 85.44 = -85.44 \text{ MPa}$

$$\begin{aligned}
 \text{Maximum shear stress, } \tau_{\max} &= \frac{\sigma_1 - \sigma_2}{2} \\
 &= \frac{-44.56 - (-85.44)}{2} \\
 &= 20.44 \text{ MPa}
 \end{aligned}$$

Ans

Problem-8: The following readings are recorded by a rectangular strain rosette (the angles are with the x-axis)

$$\epsilon_0 = 400 \times 10^{-6}, \epsilon_{45^\circ} = 200 \times 10^{-6} \text{ and } \epsilon_{90^\circ} = -100 \times 10^{-6}$$

Determine the principal strains and stresses.

Take  $E = 210 \text{ GPa}$  and Poisson's ratio 0.3.

Solution: Readings of rectangular strain rosette,

$$\epsilon_0 = 400 \times 10^{-6}, \epsilon_{45^\circ} = 200 \times 10^{-6} \text{ and } \epsilon_{90^\circ} = -100 \times 10^{-6}$$

$$E = 210 \text{ GPa}, \mu = 0.3$$

### Principal strains

For a rectangular strain rosette,

$$\epsilon_x = \epsilon_0 = 400 \times 10^{-6}, \epsilon_y = \epsilon_{90^\circ} = -100 \times 10^{-6}$$

$$\text{and } \phi = 2\epsilon_{45^\circ} - (\epsilon_x + \epsilon_y)$$

$$= 2 \times 200 \times 10^{-6} - 400 \times 10^{-6} - 100 \times 10^{-6}$$

$$= -100 \times 10^{-6}$$

$$\begin{aligned}
 \text{Principal strains, } \varepsilon_{1,2} &= \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \\
 &= \frac{10^{-6}}{2} \left[ (400 - 100) \pm \sqrt{(400 + 100)^2 + (-100)^2} \right] \\
 &= 404.95 \times 10^{-6} \text{ and } -104.95 \times 10^{-6}
 \end{aligned}$$

Principal stresses :

$$\begin{aligned}
 \sigma_1 &= \frac{E(\mu\varepsilon_2 + \varepsilon_1)}{1-\mu^2} = \frac{210000(-0.3 \times 104.95 + 404.95) \times 10^{-6}}{1 - 0.3^2} \\
 &= 86.2 \text{ MPa}, \quad \underline{\text{Ans}}
 \end{aligned}$$
  

$$\begin{aligned}
 \sigma_2 &= \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2} = \frac{210000(0.3 \times 404.95 - 104.95) \times 10^{-6}}{1 - 0.3^2} \\
 &= 3.82 \text{ MPa} \quad \underline{\text{Ans}}
 \end{aligned}$$

Problem-9: Two elastic bars are of equal length and of the same material; one is of circular cross-section of 80 mm diameter and the other of square cross-section of 80 mm side. Both absorb the same amount of strain energy under axial forces. Compare the stresses in the two bars.

Solution: First bar : circular of 80 mm diameter

Second bar : Square cross-section of 80 mm side

Equal strain energy under axial forces.

Strain energies of bars

$$\text{Strain energy of the first bar} = \frac{\sigma_1^2}{2E} \cdot AL = \frac{\sigma_1^2}{2E} \cdot \frac{\pi}{4} \times 80^2 \times L$$

$$\text{Strain energy of the second bar} = \frac{\sigma_2^2}{2E} \times 80^2 \times L$$

Equating the strain energies

$$\begin{aligned}
 \frac{\sigma_1^2}{2E} \cdot \frac{\pi}{4} \times 80^2 \times L &= \frac{\sigma_2^2}{2E} \times 80^2 \times L \\
 \Rightarrow \frac{\sigma_1}{\sigma_2} &= \sqrt{\frac{4}{\pi}} = 1.128 \quad \underline{\text{Ans}}
 \end{aligned}$$

Problem-10 : (a) A 1.5 m long steel bar has a cross-sectional area of  $800 \text{ mm}^2$ . With an elastic limit of 180 MPa, determine its proof resilience. Take  $E = 205 \text{ GPa}$ .

(b) Derive an expression for strain energy of a prismatic bar under its own weight.

Solution : (a) Given data :

$$\sigma = 180 \text{ MPa}, L = 1.5 \text{ m} = 1500 \text{ mm}$$

$$a = 800 \text{ mm}^2, E = 205 \text{ GPa} = 205000 \text{ MPa}$$

As mentioned earlier, the distinction between strain energy and resilience is hardly followed and each is referred in place of the other, i.e., it may be expressed as total strain energy per unit volume at the elastic limit.

Determination of proof resilience

$$\text{Proof resilience} = \frac{\sigma^2}{2E} \times \text{volume}$$

$$= \frac{180^2}{2 \times 20500} \times 800 \times 1500$$

$$= 94830 \text{ N-mm}$$

$$= 94.83 \text{ N-m}$$

Ans

$$\text{or proof resilience} = \frac{180^2}{2 \times 205000} = 0.079 \text{ N-mm/mm}^3$$

Ans

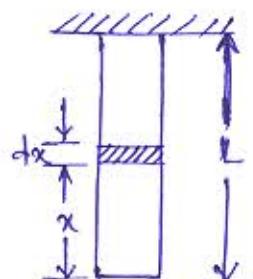
Solution : (b) Consider a bar hanging freely under its own weight. Consider a small length ' $dx$ ' of the bar at a distance ' $x$ ' from the free end.

Strain energy of elementary length

Let  $A$  = area of cross-section of the bar

$\gamma$  = specific weight i.e., weight per unit volume of the bar

$W_x$  = weight of the bar below the small section  
 $= \gamma \cdot A \cdot x$



Force applied on the elementary length is the weight of the portion of the bar below it.

$$\text{Strain energy of the element} = \frac{(\gamma \cdot A \cdot x)^2 \cdot dx}{2AE}$$

### Total strain energy

$$\begin{aligned}\text{Total strain energy} &= \int_0^L \frac{(\gamma \cdot A \cdot x)^2 \cdot dx}{2AE} \\ &= \frac{\gamma^2 \cdot A}{2E} \int_0^L x^2 \cdot dx \\ &= \frac{\gamma^2 A L^3}{6E} \quad \xrightarrow{\text{Ans}}$$

Problem-11: (a) Deduce the relation,  $E = \frac{9KG}{3K+G}$

(b) A bar of 24 mm diameter and 400 mm length is acted upon by an axial load of 38 kN. The elongation of the bar and the change in diameter are measured as 0.165 mm and 0.0031 mm respectively. Determine

- i) the Poisson's ratio
- ii) the values of the three moduli

Solution: (a) We know,

$$E = 2G(1+\mu)$$

$$\Rightarrow 1+\mu = \frac{E}{2G}$$

$$\Rightarrow 2+2\mu = \frac{E}{G} \quad \text{--- (1)}$$

Also,

$$E = 3K(1-2\mu)$$

$$\Rightarrow 1-2\mu = \frac{E}{3K} \quad \text{--- (2)}$$

Adding (1) & (2),

$$3 = E \left( \frac{1}{G} + \frac{1}{3K} \right)$$

$$\Rightarrow 3 = \frac{E}{3KG} (3K+G)$$

$$\Rightarrow \boxed{E = \frac{9KG}{3K+G}}$$

(proved)

(b) Given data : A circular bar with

$d = 24 \text{ mm}$ ,  $L = 400 \text{ mm}$   
 $P = 38000 \text{ N}$ ,  $\Delta L = 0.165 \text{ mm}$   
 $\Delta d = 0.0031 \text{ mm}$

$$A = \frac{\pi}{4} \times (24)^2 = 144\pi \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{38000}{144\pi} = 84 \text{ MPa}$$

Poisson's ratio ( $\mu$ ) :

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\left(\frac{\Delta d}{d}\right)}{\left(\frac{\Delta L}{L}\right)}$$

$$\Rightarrow \frac{\Delta d}{d} = \mu \cdot \frac{\Delta L}{L}$$

$$\Rightarrow \frac{0.0031}{24} = \mu \cdot \frac{0.165}{400}$$

$$\Rightarrow \boxed{\mu = 0.313}$$

Ans

Elastic Moduli (E, G & K)

$$E = \frac{\sigma}{\epsilon} = \frac{84}{\left(\frac{0.165}{400}\right)} = 203636 \text{ MPa}$$

Ans

$$\text{Also, } E = 2G(1+\mu)$$

$$\Rightarrow G = \frac{E}{2(1+\mu)} = \frac{203636}{2(1+0.313)} = 77546 \text{ MPa}$$

Ans

$$\text{Again, } E = 3K(1-2\mu)$$

$$\Rightarrow K = \frac{E}{3(1-2\mu)} = \frac{203636}{3(1-2 \times 0.313)} = 181494 \text{ MPa}$$

Ans